Physical Sciences

Novel Concise Robust Control Design for Non-square Systems with Multiple Time Delays

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In order to improve the robustness of industrial non-square systems with multiple time delays, contrary to omitting the time delays as a model perturbation, a reformative closed-loop gain shaping algorithm (CGSA) was presented using Padé stable approximation of time delays. The algorithm is more precise and concise to design the robust controller. The simulation results show that the robust controller can keep the set inputs without overshoot and steady state error, the results are satisfactory. And the control effects are a little better than those in previous papers. *Journal of Nature and Science, 1(2):e41, 2015.*

Robust | closed-loop gain shaping | non-square systems | model perturbation | Padé approximation

Non-square system is a common industrial process in fields of the practical engineering. The number of the input variables does not equal to that of the outputs, e.g. the steam generator level control system, the fuel cell voltage control system [1, 2], etc. The control method of a non-square system is to transform it into a square system by adding or deleting the input and output variables. But this conventional method can not only increase the control cost but also reduce the control performance, so it is difficult to obtain a satisfactory result.

The method of decoupling internal model control for multi-variable non-square system with time delays was proposed in [3], the controller had good performance of tracking ability and strong robustness, but as the traditional internal model control if the model was mismatched, the control effects would be variation and the simulation result had overshoot also. A modified internal model control method for non-square systems was proposed in [4], the control effect of the method was satisfactory. A controller was designed by steady gain matrix of non-square systems in [5]. However the method belonged to the static decoupling method which needed to regulate two parameters and it was time-consuming. A PI control method for non-square systems was proposed in [6], the control effect of the method was dissatisfactory. In [3~6], the designed controllers had all pure time delays and were not minimum phase systems. Some of them had zeros in the right-half plane.

The author presented a kind of closed-loop gain shaping algorithm (CGSA) to non-square systems with multiple time delays in 2009. The robust control design algorithm for non-square systems was given through solving the pseudo-inverse of the nominal plant without multiple time delays [7, 8]. It has a simple solving process. The controller designed by this algorithm has lower order and good robustness. CGSA has been applied in many different fields [9-12].

Contrary to omitting the time delays, this paper will study the system further, using Padé stable approximation and equivalenting the time delays as the first order component. Then it will obtain the robust controller with closed-loop gain shaping algorithm. The new robust controller can keep the set inputs without overshoot and steady state error, and it has good robustness. The control effects are a little better than those in [7], which are further verified the feasibility and validity.

Closed-Loop Gain Shaping Algorithm (CGSA)

Inspired by H_∞ robust control and loop shaping theory, CGSA is presented based on understanding of H_∞ mixed sensitivity functions and rich experience of designing controller as well as on the correlativity between the sensitivity function S and the complementary sensitivity function T (T=I-S). Therefore CGSA can be considered as a simplified H_∞ robust control algorithm from the viewpoint of engineering implementation. The tedious mathematical deduction is replaced by physical understanding in CGSA. The advantage of CGSA is that the physical meanings supporting the idea are clear and the solving procedure is relatively simple, the robust performance and robust stability of the system are satisfactory.

CGSA uses the result of the mixed sensitivity algorithm of H_∞ robust control theory. It constructs directly the complementary sensitive function T using four parameters with engineering sense and constructs indirectly the sensitive function S because of the correlativity between S and T, then the controller K is reversely deduced out. If we say that the H_∞ mixed sensitivity algorithm is the result of positive thinking and the loop shaping theory can be considered as the result of divergent thinking, however CGSA is the result of reverse thinking. The core of the algorithm is to construct T using four parameters with engineering sense such as bandwidth frequency, high frequency asymptote slope, the largest singular value and the peak value of the closed-loop frequency spectrum, and construct indirectly the sensitive function S. So the robust performance and robust stability of the systems are guaranteed. The advantage of CGSA is that the physical meanings supporting the idea are clear and the solving procedure is relatively simple.

For signal tracking of MIMO systems, the closed-loop transfer function matrix of system from input to output is actually the complementary sensitive matrix T

$$\boldsymbol{GK}(\boldsymbol{I}+\boldsymbol{GK})^{-1}=\boldsymbol{T}$$

From the formula above, we obtain

$$\boldsymbol{K}=\boldsymbol{G}^{-1}(\boldsymbol{I}-\boldsymbol{T})^{-1}\boldsymbol{T} \qquad (1)$$

For MIMO systems where G is a square matrix, let every element of T be first-order or second-order or third-order inertia component with the largest singular value of 1. For MIMO non-square systems with dimension greater than 2, the formula is slightly complex due to involving generalized inverse problem.

The definition of generalized inverse is

$$\boldsymbol{G}^{-1}=\boldsymbol{G}^{\mathrm{T}}(\boldsymbol{GG}^{\mathrm{T}})^{-1}$$

For the closed-loop system with two inputs and two outputs, let the non-diagonal elements of T be zeros and the diagonal elements of T be the first-order inertia components with the largest singular value of 1. We can write

$$\boldsymbol{T}=\begin{bmatrix}\dfrac{1}{T_{11}s+1} & 0\\ 0 & \dfrac{1}{T_{22}s+1}\end{bmatrix}$$

i.e. the two inputs and the two outputs are completely decoupled.

Let G be a 2×3 non-square matrix.

$$\boldsymbol{G}=\begin{bmatrix}G_{11} & G_{12} & G_{13}\\ G_{21} & G_{22} & G_{23}\end{bmatrix}$$

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Then we can obtain the equation (2).

$$\boldsymbol{G}^{-1}=\frac{\begin{bmatrix} G_{11}(G_{21}^2+G_{22}^2+G_{23}^2)-G_{21}(G_{11}G_{21}+G_{12}G_{22}+G_{13}G_{23}) & G_{21}(G_{11}^2+G_{12}^2+G_{13}^2)-G_{11}(G_{11}G_{21}+G_{12}G_{22}+G_{13}G_{23}) \\ G_{12}(G_{21}^2+G_{22}^2+G_{23}^2)-G_{22}(G_{11}G_{21}+G_{12}G_{22}+G_{13}G_{23}) & G_{22}(G_{11}^2+G_{12}^2+G_{13}^2)-G_{12}(G_{11}G_{21}+G_{12}G_{22}+G_{13}G_{23}) \\ G_{13}(G_{21}^2+G_{22}^2+G_{23}^2)-G_{23}(G_{11}G_{21}+G_{12}G_{22}+G_{13}G_{23}) & G_{23}(G_{11}^2+G_{12}^2+G_{13}^2)-G_{13}(G_{11}G_{21}+G_{12}G_{22}+G_{13}G_{23}) \end{bmatrix}}{(G_{11}^2+G_{12}^2+G_{13}^2)(G_{21}^2+G_{22}^2+G_{23}^2)-(G_{11}G_{21}+G_{12}G_{22}+G_{13}G_{23})^2} \tag{2}$$

Therefore, the controller (3) is as follows.

$$\boldsymbol{K}=\boldsymbol{G}^{-1}(\boldsymbol{I}-\boldsymbol{T})^{-1}\boldsymbol{T}=\boldsymbol{G}^{-1}\begin{bmatrix}\frac{T_{11}s}{T_{11}s+1} & 0\\ 0 & \frac{T_{22}s}{T_{22}s+1}\end{bmatrix}^{-1}\begin{bmatrix}\frac{1}{T_{11}s+1} & 0\\ 0 & \frac{1}{T_{22}s+1}\end{bmatrix}=\boldsymbol{G}^{-1}\begin{bmatrix}\frac{1}{T_{11}s} & 0\\ 0 & \frac{1}{T_{22}s}\end{bmatrix}=$$

$$\frac{\begin{bmatrix}(G_{11}(G_{21}^2+G_{22}^2+G_{23}^2)-G_{21}(G_{11}G_{21}+G_{12}G_{22}+G_{13}G_{23}))/T_{11} & (G_{21}(G_{11}^2+G_{12}^2+G_{13}^2)-G_{11}(G_{11}G_{21}+G_{12}G_{22}+G_{13}G_{23}))/T_{22}\\ (G_{12}(G_{21}^2+G_{22}^2+G_{23}^2)-G_{22}(G_{11}G_{21}+G_{12}G_{22}+G_{13}G_{23}))/T_{11} & (G_{22}(G_{11}^2+G_{12}^2+G_{13}^2)-G_{12}(G_{11}G_{21}+G_{12}G_{22}+G_{13}G_{23}))/T_{22}\\ (G_{13}(G_{21}^2+G_{22}^2+G_{23}^2)-G_{23}(G_{11}G_{21}+G_{12}G_{22}+G_{13}G_{23}))/T_{11} & (G_{23}(G_{11}^2+G_{12}^2+G_{13}^2)-G_{13}(G_{11}G_{21}+G_{12}G_{22}+G_{13}G_{23}))/T_{22}\end{bmatrix}}{((G_{11}^2+G_{12}^2+G_{13}^2)(G_{21}^2+G_{22}^2+G_{23}^2)-(G_{11}G_{21}+G_{12}G_{22}+G_{13}G_{23})^2)s} \tag{3}$$

Controller Design

Taking a 2×3 non-square matrix in [4] as an example, the transfer function matrix of the system model is

$$\boldsymbol{G}(s)=\begin{bmatrix}\frac{4.05\mathrm{e}^{-81s}}{50\mathrm{s}+1} & \frac{1.77\mathrm{e}^{-84s}}{60\mathrm{s}+1} & \frac{5.88\mathrm{e}^{-81s}}{50\mathrm{s}+1}\\ \frac{5.39\mathrm{e}^{-54s}}{50\mathrm{s}+1} & \frac{5.72\mathrm{e}^{-42s}}{60\mathrm{s}+1} & \frac{6.9\mathrm{e}^{-45s}}{40\mathrm{s}+1}\end{bmatrix} \tag{4}$$

The controller was given by using the modified internal model control (IMC) method in [4]

$$\boldsymbol{Q}(s)=\begin{bmatrix}\frac{-0.183s+0.0791}{67.69s+1} & \frac{5.2959s-0.0032}{62.946s+1}\mathrm{e}^{-3s}\\ \frac{-18.467s-0.336}{8.7374s+1}\mathrm{e}^{-12s} & \frac{15.252s+0.2751}{4.9648s+1}\\ \frac{15.482s+0.2167}{14.925s+1}\mathrm{e}^{-9s} & \frac{-9.6823s-0.0806}{31.575s+1}\mathrm{e}^{-3s}\end{bmatrix}$$

The controller has pure time delays and zeros in the right-half plane, and it is not a minimum phase system.

This paper uses Padé approximation to linearize the time delay system more precisely, if the time delay link $e^{-\tau s}$ can be approximated by rational transfer function, then the typical Padé approximation of order n can be expressed as [13]

$$e^{-\tau s}\approx\frac{1-\tau s/2+p_1(\tau s)^2-p_2(\tau s)^3+\cdots}{1+\tau s/2+p_1(\tau s)^2-p_2(\tau s)^3+\cdots} \tag{5}$$

where the second approximation $p_1=1/12$, the third approximation $p_1=1/10, p_2=1/120$.

Equation (5) shows that numerator and denominator have the same order in the approximate formula, but the numerator polynomial has negative coefficient, which can cause system unstable when the closed-loop system is constructed. In order to solve this problem we can derive more proper Padé approximate formula.

The Taylor series of time delay function $e^{-\tau s}$ can expand and preserve to the first order, which can be expressed:

$$e^{-\tau s}\approx 1-\tau s$$

using the advanced mathematics' formula $(1+x)^{1/n}\approx 1+\frac{1}{n}x$, when $x\to 0$, let $n=-1, x=\tau s$, then

$$e^{-\tau s}\approx\frac{1}{1+\tau s} \tag{6}$$

then

$$G_{11}^{'}=\frac{4.05}{(50s+1)(81s+1)},\quad G_{12}^{'}=\frac{1.77}{(60s+1)(84s+1)},$$

$$G_{13}^{'}=\frac{5.88}{(50s+1)(81s+1)},\quad G_{21}^{'}=\frac{5.39}{(50s+1)(54s+1)},$$

$$G_{22}^{'}=\frac{5.72}{(60s+1)(42s+1)},\quad G_{23}^{'}=\frac{6.9}{(40s+1)(45s+1)},$$

if these parameter calculation is used to design the controller directly, the final controller's form is complex, which has many parameters and large amount of calculation. In order to simplify the problem and reduce the calculation, making the time delay in every row of the controlled object in (4) equal to its maximum value, then

$$G_{11}^{'}=\frac{4.05}{(50s+1)(84s+1)},\quad G_{12}^{'}=\frac{1.77}{(60s+1)(84s+1)},$$

$$G_{13}^{'}=\frac{5.88}{(50s+1)(84s+1)},\quad G_{21}^{'}=\frac{5.39}{(50s+1)(54s+1)},$$

$$G_{22}^{'}=\frac{5.72}{(60s+1)(54s+1)},\quad G_{23}^{'}=\frac{6.9}{(40s+1)(54s+1)}.$$

Let $G'=\begin{bmatrix}G_{11}^{'} & G_{12}^{'} & G_{13}^{'}\\ G_{21}^{'} & G_{22}^{'} & G_{23}^{'}\end{bmatrix}, G_0=\begin{bmatrix}\frac{1}{84s+1} & 0\\ 0 & \frac{1}{54s+1}\end{bmatrix}$

Then it has

$$\begin{aligned}\mathbf{G}'^{-1}&=(\mathbf{G}_0\mathbf{G})^{\mathrm{T}}(\mathbf{G}_0\mathbf{G}(\mathbf{G}_0\mathbf{G})^{\mathrm{T}})^{-1}=\mathbf{G}^{\mathrm{T}}\mathbf{G}_0^{\mathrm{T}}(\mathbf{G}_0\mathbf{G}\mathbf{G}^{\mathrm{T}}\mathbf{G}_0^{\mathrm{T}})^{-1}\\&=\mathbf{G}^{\mathrm{T}}\mathbf{G}_0^{\mathrm{T}}(\mathbf{G}_0^{\mathrm{T}})^{-1}(\mathbf{G}\mathbf{G}^{\mathrm{T}})^{-1}\mathbf{G}_0^{-1}=\mathbf{G}^{-1}\mathbf{G}_0^{-1}\end{aligned}$$

that the new matrix inversion equals original matrix inversion multipliy by the changed inverse matrix, so the solving of the new robust controller will change into a simple process, that is

$$\boldsymbol{K}' = \boldsymbol{K}\boldsymbol{G}_0^{-1}$$

where K is the robust controller in [7].

The final designed controller is a sixth order controller. The third order controller can be acquired through a new order-reduction algorithm based on the stability of consideration, which took advantage of the approximate method of the Routh [13]. The reduced third-order controller is given in equation (7).

$$\boldsymbol{K}' = \begin{bmatrix} \dfrac{(1.4753e-3s+4.7266e-6)*(84s+1)}{T_{11}s(s^2+1.3706e-2s+5.9772e-5)} & \dfrac{(-1.6383e-4s-1.8861e-7)*(54s+1)}{T_{22}s(s^2+1.3706e-2s+5.9772e-5)} \\ \dfrac{(-6.0819e-3s-2.0081e-5)*(84s+1)}{T_{11}s(s^2+1.3706e-2s+5.9772e-5)} & \dfrac{(4.8670e-3s+1.6440e-5)*(54s+1)}{T_{22}s(s^2+1.3706e-2s+5.9772e-5)} \\ \dfrac{(3.5933e-3s+1.2954e-5)*(84s+1)}{T_{11}s(s^2+1.3706e-2s+5.9772e-5)} & \dfrac{(-1.3027e-3s-4.8190e-6)*(54s+1)}{T_{22}s(s^2+1.3706e-2s+5.9772e-5)} \end{bmatrix} \quad (7)$$

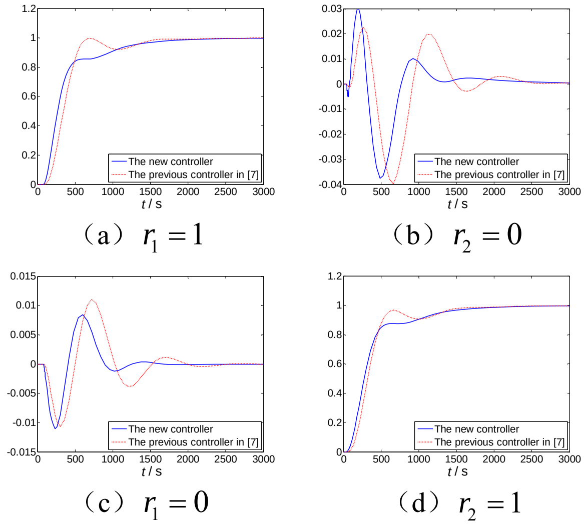

Fig.1.Set-point responses

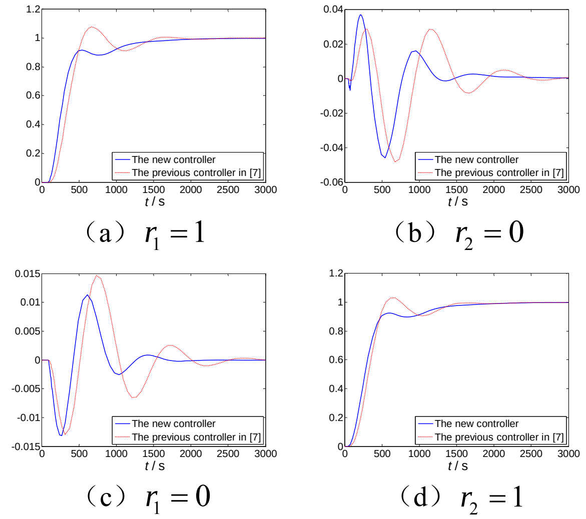

Fig.3.Model perturbation responses (2)

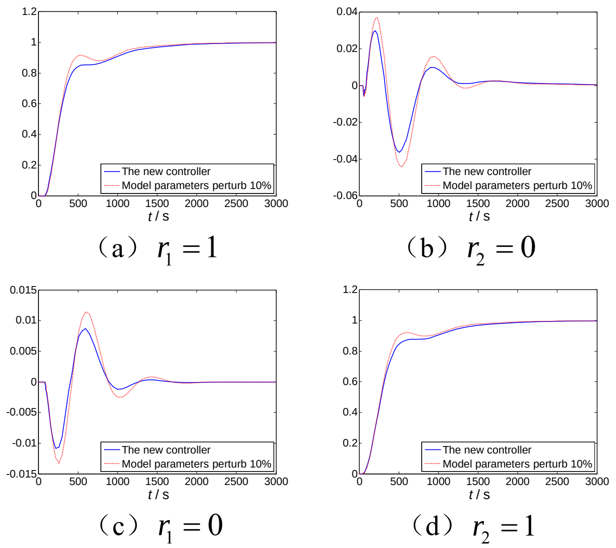

Fig.2.Model perturbation responses (1)

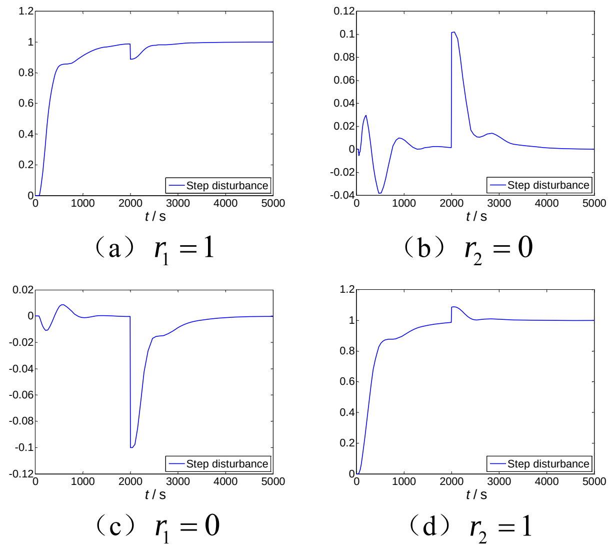

Fig.4.Disturbance responses

Simulation Analysis

The simulation results of set-point response are given in Fig.1, where $T_{11}=400$, $T_{22}=400$. Two output curves correspond to system inputs of $[1\ 0]^T$ and $[0\ 1]^T$ respectively. The solid line is the new controller (7), and the dotted line is the previous controller in [7]. From the analysis of Fig.1, it follows that the system regulating time is about 1300s and the new controller can keep the set inputs without overshoot and static error, and the control effects of the new controller (7) are a little better than those in [7].

The simulation results of model perturbation response (1) are given in Fig.2, where the gains, the time delays and the time constants of every element in the transfer function matrix are increased by 10%. The solid line is the unchanged transfer function, and the dotted line is the changed transfer function. From the analysis of Fig.2, it follows that the system control effect is satisfactory without overshoot and static error, and the new controller has good robustness.

The simulation results of model perturbation response (2) are given in Fig.3, where the gains, the time delays and the time constants of every element in the transfer function matrix are increased by 10%. The solid line is the new controller, and the dotted line is the previous controller in [7]. From the analysis of Fig.3, it follows that the previous controller has overshoot and the new controller has a better robustness.

The simulation results of disturbance responses are given in Fig.4, where we set step disturbance for the first channel with the magnitude of -0.1 and for the second channel with 0.1 at 2000s respectively. From the analysis of Fig.4, it follows that the new controller can get satisfactory results even there is a step disturbance in the system, and the new controller has good robustness.

Conclusions

CGSA for multiple and large time delays non-square systems is proposed in this paper using Padé stable approximation of time delays, its control performance is satisfactory. The control effects are a little better than those in [7]. Compared with [3-6], the designing procedure of controller is relatively simple. The designed controller in this paper has not pure time delays and zeros in the right-half plane, and is a stable minimum phase system. The simulation results show that this method has better control performance and good robustness than other control methods of non-square systems.

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